LITERATURE CITED

1. E. Sternberg and R. Muki, "The effect of couple-stresses on the stress concentration around a crack," Intern. J. Solids Struct., 3, 69-95 (1967).
2. G. Tiwari, "Effect of couple-stresses on the distribution of stress along a crack," J. Sci. Eng. Res., 12, 125-138 (1968).
3. H. Hahn, "Rissprobleme im Cosserat-Kontinuum," Z. Angew Math. Mech., 51, 143-145 (1971).
4. V. Z. Parton and B. A. Kudryavtsev, "On a method of solving problems about a crack and a stamp in couple-stress elasticity theory," Probl. Prochn., No. 3, 78-81 (1971).
5. S. Itou, "The effect of couple-stresses on the stress concentration around an elliptical hole," Acta Mech., 16, 289-296 (1973).
6. N. I. Muskhelishvili, Some Fundamental Problems of Mathematical Elasticity Theory [in Russian], 5th ed., Nauka, Moscow (1966).
7. R. Mindlin, "Complex representation of displacements and stresses in plane strain with couple-stresses," in: Transactions of an International Symposium in Tbilisi [in Russian], Vol. 1, Nauka, Moscow (1963).
8. G. Irwin, "Fracture," in: Handbuch der Physik, Vol. 6, Springer-Verlag, Berlin (1958), pp. 551-590.
9. Ya. M. Shiryaev, "Stress concentration near an inhomogeneity and experimental clarification of the effect of couple-stresses," Zh. Prikl. Mekh. Tekh. Fiz., No. 4, 142-144 (1976).

COMPRESSIVE STRENGTH OF MATERIALS

M. M. Muzdakbaev and V. S. Nikiforovskii

UDC 539.4

A state of compression in material, rocks, machine parts, and structural elements is evidently very common. As a measure of this state, the concept of uniaxial compressive strength was introduced and considered as a fundamental characteristic of the material. One might also suppose that a material is fractured when the maximum tangential stress reaches the breaking point, which turns out to be half as large as the compressive strength. It should be noted that the compressive strength depends rather strongly on many factors, including the shape of the sample, its dimensions and volume, and end conditions [1, 2]. Experimenters long ago concluded that the uniaxial compressive strength is not a characteristic of the material [3, 4]. Actually, the description of the fracture patterns of samples recorded so far [2, 3, 5] with the formation of oblique fracture surfaces or surfaces of discontinuity by a coaxial compressive load can more probably be related to the shear or tensile stresses than to the compressive load. In addition, experimental and theoretical studies have shown that in tests which at first glance seem simple, the pattern of the stressed state is complex, not one-dimensional, and varies with the experimental conditions [6-9].

A number of results were obtained in [9] from a study of the stressed state of a sample under plain strain by investigating the effects of dimensions, the correlation of properties, and the role of the inserts.

In the present paper we present a numerical study of the state of stress of tubular samples and consider certain characteristic features which are interesting and important from the point of view of understanding the significance of compressive strength. The calculation was performed by the method of finite elements, using triangular-shaped elements [10]. In a cross section of a tubular sample along a meridional plane shown in Fig. 1, D and d are the outside and inside diameters, L is the length of the sample, and L_1 is the length of the inserts. The division into elements is shown in the upper symmetric half ABFE. The stressed state is characterized by the four components of the stress tensor σ_z, σ_r, σ_θ, and τ_{rz}. Since the pattern is symmetric with respect to a quarter of the cylinder (OO' is the axis of symmetry and AB is the plane of symmetry), we set up and solve the problem for the region ABFE. We formulate the following boundary conditions:

$$\begin{gathered} v=0,\ \tau_{rz}=0;\ z=L/2,\ d/2<r<D/2; \\ u=0,\ v=\text{const};\ z=L,\ d/2<r<D/2; \\ \tau_{rz}=0,\ \sigma_r=0;\ r=d/2,\ D/2,\ L/2<z<L. \end{gathered} \tag{1}$$

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 2, pp. 154-160, March-April, 1978. Original article submitted April 29, 1977.

0021-8944/78/1902-0262$07.50

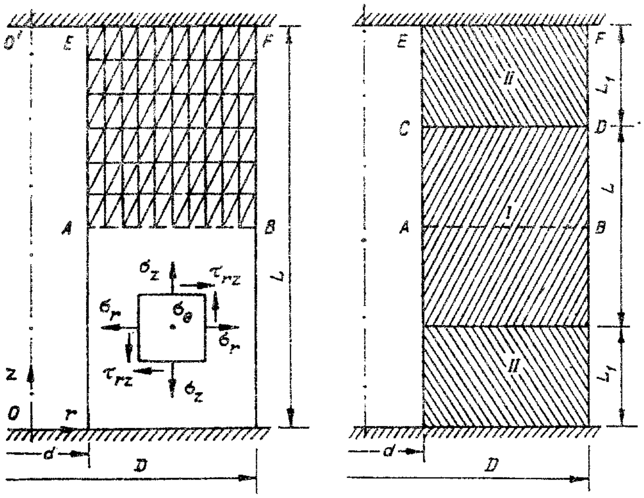

Fig. 1

The first of boundary conditions (1) denote the conditions in the plane of symmetry $z = L/2$ over the whole thickness of the sample. At the upper end $z = L$ the displacement $v = \text{const}$, $u = 0$ is in the vertical direction only; here u and v are, respectively, the horizontal and vertical components of the displacement of points. The lateral faces are stress-free. It should be noted that specifying no horizontal displacements of points on the upper boundary between the sample and the pressure plate is a limiting case. Actually, there is some slippage. Nevertheless, this intentionally crude formulation of the problem enables us to see and understand the effect of the nonuniformity of the pattern more quickly.

Figure 2 shows the stress field in samples with the diameter of the base D equal to the length L, $D/d = 3.7$, and Young's modulus $E = 5 \cdot 10^5$ kg/cm^2 and Poisson's ratio $\nu = 0.15$ for the material (marble). The curves are isobars – lines of equal stress. The solid curves indicate compressive, the dashed tensile, and the dashed-dot curves zero values of the stress components σ_z^0, σ_r^0, σ_θ^0, τ_{rz}^0, and τ_{max}^0 as percentages of the average vertical stress $\sigma_{ij}^0 = \sigma_{ij}\,100/\bar{\sigma}_z$; the region of normal tensile stresses is shaded. The nonuniformity of the stressed state appears in all components. Thus, the vertical stress σ_z is somewhat larger on the outside diameter and smaller on the inside than the average stress near the end; in the central part the pattern is opposite, with the stress being larger on the inside and smaller on the outside. The radial σ_r and azimuthal σ_θ normal components and the tangential τ_{rz} component of the stress tensor are nonzero. Two features are noted: a) The concentrations of all these stresses occur near the end in the vicinity of the bounding surface; b) there is a change in sign of the normal stresses in the central part of the sample and an appreciable gradient of the tangential component near the end at the outside diameter.

These two facts must be noted in analyzing the possible fracture mechanism. Study of the isobars shows that fracture in vertical planes coaxial with the compression is really not paradoxical as noted in [11]; there is a real reason – elongation in the radial and azimuthal cross sections. On the diagram of the isobars of the maximum tangential stress

$$|\tau_{max}| = \frac{1}{2}\sqrt{(\sigma_r - \sigma_z)^2 + 4\tau_{rz}^2}$$

there is a region of concentration and large gradients near the end at the outside diameter. The pattern is nonuniform for all components over the whole volume of the sample.

The effect of the ends on one another decreases as the vertical dimension is increased; the end zones break up and cease affecting one another. Localization of the zones ends for $L \approx 2D$, and simultaneously with this it is important to note the following features (Fig. 3, $L = 2D$, $D/d = 3.7$, $E = 5 \cdot 10^5$ kg/cm^2, $\nu = 0.15$). The end zone occupies only the upper part of the pattern; in the middle part the vertical component is close to the average value, the radial and azimuthal normal components and the tangential component are close to zero, with the radial and azimuthal stresses being weakly positive (tensile) in a broad central zone; the maximum tangential stress in the uniform zone is close to half the average value of the vertical stress, as it should be. There is a certain increase in the nonuniformity of the pattern in the end zone; the σ_r, σ_θ, τ_{rz}, and particularly the τ_{max} stress concentrations are increased, if only slightly.

It is known from experiments that the limiting value in uniaxial compression is decreased and stabilized as the vertical dimension is increased. Many relate this stabilization for $L \gtrsim 2D$ to the appearance of a zone

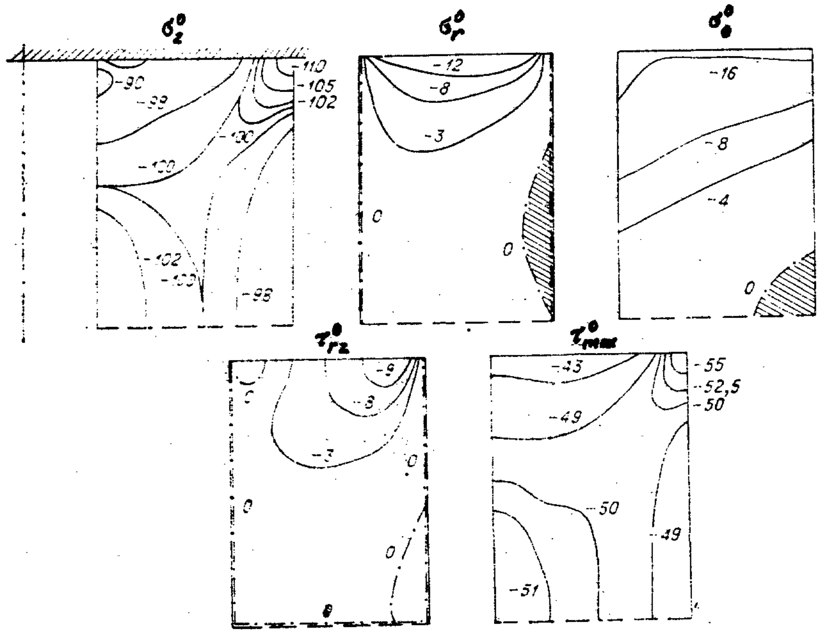

Fig. 2

of uniform stress. As noted above, as L is increased from D to 2D, end zones appear, break up, and cease to affect one another; a uniform zone is organized, and at first glance the desired result is obtained by a change in geometry. However, a simultaneous insignificant increase in the concentration of τ_{max}, the formation of extensive zones of normal tensile stresses, and the experimental result of the fracture of samples from the plane of the pressure plates from the outside region, i.e., from the region of nonuniform stress or along vertical planes, also are the cause of a weak nonuniformity in the central part. From these comparisons we can conclude that the tendency toward uniformity of the pattern has not been confirmed: the decrease in the breaking strength can be related to the increase in the concentration of τ_{max} and the change in the nature of the fracture with the appearance of elongation in vertical cross sections as a new cause. A decrease in D/d leads to the same result as an increase in the length of the sample.

One way of decreasing the end effect and obtaining a uniform stress field is to reduce friction on the ends by using a lubricant appropriate to the shape and properties of the inserts. In using tubular steel inserts having the shape of the sample, the uniformity of the stressed state of the sample is actually increased, as shown in Fig. 4, where $L/D = 2$, $L_1/D = 1$, $D/d = 3.7$, $E_1 = 5 \cdot 10^5$ kg/cm^2, $E_2 = 2.1 \cdot 10^6$ kg/cm^2, $\nu_1 = 0.15$, and $\nu_2 = 0.27$. The inserts are shown as regions II of Fig. 1b; AB is the plane of symmetry and OO' is the axis of symmetry. The finite elements are decreased in size on the surface of separation CD between the sample and the insert just as in the end zone. The tendency, noted above, for the stress field to change with an increase in length of the sample is observed. In this situation the stressed state in the sample is practically uniform; the vertical component is close to the average value; the radial and azimuthal normal components and the tangential component of the stress field are close to zero except in a narrow zone close to the contact with the insert. In this zone τ_{max} is nonuniform, and in the insert itself the situation is as described above with a formed end zone in which the stress concentration is somewhat higher than in the adjacent zone. This must be related to the larger Poisson's ratio (0.27 instead of 0.15) rather than to the tendency to increase with an increase in the length of the sample. Different cases are possible on the sample–insert boundary, since for the same deformation of different materials differences in the radial ε_r and azimuthal ε_θ strains appear because of different values of Young's modulus and Poisson's ratio.

A very interesting situation is observed when the inserts are soft and easily deformed. In this case the sample generally does not fracture along slip planes, but along vertical planes parallel to the direction in which the load acts. The action of such an insert is opposite that for a rigid fastening. Since the insert is deformed more strongly in the transverse direction it will drag the sample along with it by friction and give rise to elongation. The insert itself will be appreciably compressed. Figure 5 shows patterns of isobars for $L = D$, $L_1 = 0.05L$, $D/d = 3.7$, $E_1 = 5 \cdot 10^5$ kg/cm^2, $E_2 = 5 \cdot 10^3$ kg/cm^2, $\nu_1 = 0.15$, and $\nu_2 = 0.45$ (cf. Fig. 1, where region II is lead). The qualitative changes of all the components of the stress tensor are particularly noteworthy. Thus, the zones in Fig. 2 where the stresses are below and above the average value of the vertical component have exchanged places. The signs of the radial and azimuthal normal components and the tangential

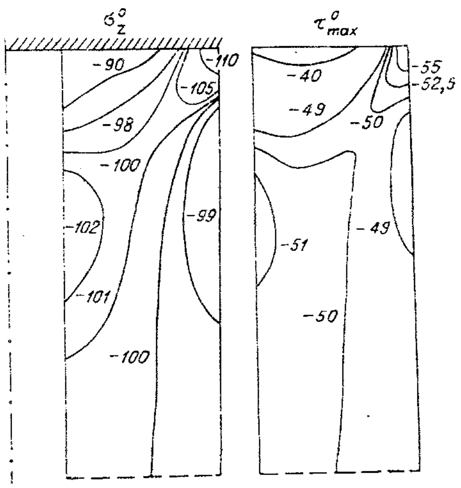

Fig. 3

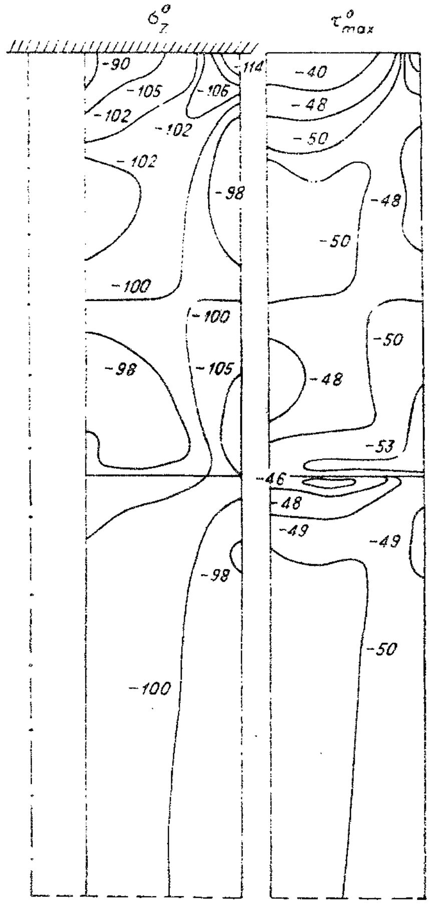

Fig. 4

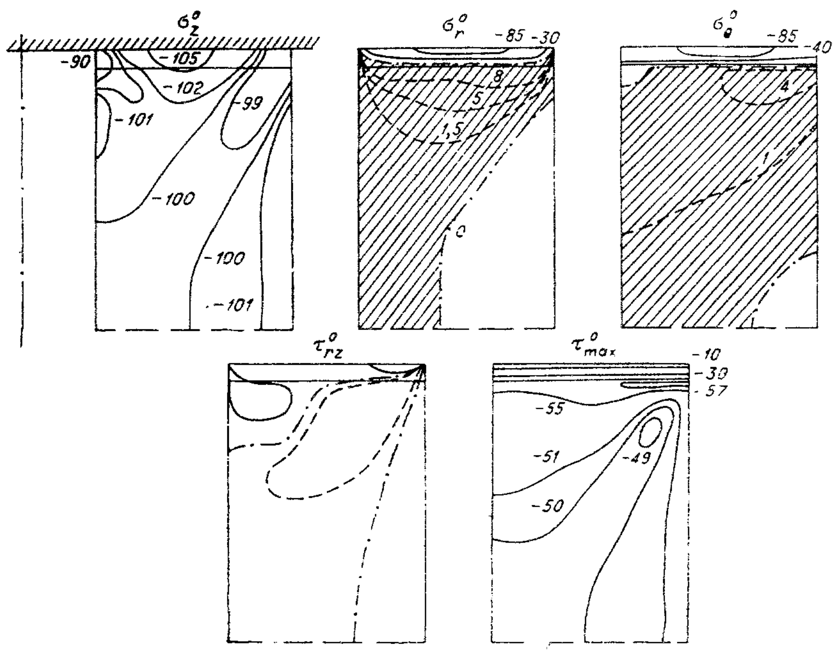

Fig. 5

component are also changed. The concentration of maximum tangential stress has increased. While the concentration of τ_{max} increased only slightly, the zone where σ_r and σ_θ are tensile stresses now occupies practically the whole sample; the amplitudes, although relatively small, have increased several times. The transition from one form of fracture to another is therefore quite natural.

Our calculations and considerations show that a simple experiment on uniaxial compression turns out to be rather complicated from the point of view of analysis. The stressed state of a sample is essentially nonuniform, and the fracture conditions are satisfied first in the nonuniform region of the pattern near the pressure plates of the testing machine. As a result the uniaxial compressive strength is a convenient technical strength characteristic of a structural sample rather than a characteristic of the material.

LITERATURE CITED

1. Yu. M. Kartashev and A. A. Grokhol'skii, Operating Instructions for Determining the Compressive Strength of Rocks [in Russian], VNIMI, Leningrad (1973).
2. E. I. Il'nitskaya, Properties of Rocks and Methods of Determining Them [in Russian], Nedra, Moscow (1969).
3. G. N. Kuznetsov, Mechanical Properties of Rocks [in Russian], Ugletekhizdat, Moscow (1947).
4. L. S. Burshtein, Static and Dynamic Tests of Rocks [in Russian], Nedra, Leningrad (1970).
5. B. Paul, "Macroscopic criteria of plastic flow and brittle fracture," in: Fracture [Russian translation], Vol. 2, Mir, Moscow (1975).
6. B. T. Brady, "A mechanical equation of state for brittle rock," Intern. J. Rock Mech. Min. Sci., 7, 385 (1970); 10, 281-309 (1973).
7. E. T. Brown, "Controlled failure of hollow rock cylinders in uniaxial compression," Rock Mech., 4, 1-24 (1972).
8. M. Al-Chalabi and C. L. Huang, "Stress distribution within circular cylinders in compression," Intern. J. Rock Mech. Min. Sci., 11, No. 2 (1974).
9. R. B. Beisetaev and V. S. Nikiforovskii, "Strength of solids under uniaxial compression," Fiz.-Tekh. Probl. Razrab. Polezn. Iskop., No. 3, 15-20 (1976).
10. O. Zenkevich and I. Chang, The Method of Finite Elements in the Theory of Structure and the Mechanics of Continuous Media [in Russian], Nedra, Moscow (1974).
11. P. W. Bridgman, Studies in Large Plastic Flow and Fracture, Harvard University Press, Cambridge (1954).

CONVECTIVE EFFECTS IN LIQUID INCLUSIONS DRIFTING IN NONUNIFORMLY HEATED SOLIDS

Yu. K. Bratukhin

UDC 548.5:536.2

§1. We will consider a liquid-filled spherical cavity in an infinite solid mass. The liquid dissolves the surrounding material and under equilibrium conditions is a saturated solution of concentration C_0. At infinity let there be a constant horizontal temperature gradient $\nabla T_e = \mathbf{A}$. Under these conditions in the gravitational field $\mathbf{g}$ free convective motion develops in the liquid.

We assume that the motion is slow and steady; solid phase can crystallize out of the supersaturated solution only at the interface between the inclusion and the matrix; the dissolving of the solid in the liquid does not lead to a change in the volume of the latter; the thermal diffusion and diffusion heat-conduction effects are negligible [1]. All the parameters (kinematic and dynamic viscosity coefficients ν and η, thermal conductivity $\varkappa$, thermal diffusivity χ, and diffusion coefficient D) of the liquid and the solid are constant. The solubility C_0 and the liquid density ρ depend linearly on temperature T. We assume that the density also depends on the concentration C, defined as the ratio of the mass of solid material per unit volume of solution to the mass of that volume:

$$\rho(T,\ C) = \rho(T_0,\ C_0)\,[1 + \alpha(C - C_0) - \beta(T - T_0)],$$

$$C_0(T) = C_0(T_0) + (dC_0/dT)(T - T_0).$$

The nonuniform heating of the walls of the cavity leads to the dissolving of the hotter parts of the solid and subsequent diffusive and convective mass transfer to the cooler regions, where the solution is supersaturated

Perm'. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 2, pp. 160-166, March-April, 1978. Original article submitted March 15, 1977.